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OPTIMUM CONTROL OF POROUS BODY HEATING BY A FLOW OF INCOMPRESSIBLE LIQUID (GAS)

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Optimum control of the process of heating of a porous body by a flow of an incompressible liquid or gas is considered. The amount of heat energy accumulated in the porous body is selected as an optimization criterion. This amount must be maximized provided there are a prescribed amount of heat that can be transferred to the flow filtering through the porous body and prescribed duration of the process. The control is considered to be exercised by means of the temperature of the liquid (gas) flow on entry into the porous body.

Accumulation of heat energy is one of the important directions in the application of porous media in power systems [1]. Calculation and optimization of heat- and mass-transfer processes in porous bodies are important for increasing the efficiency of such systems. It should be noted that in the foreign literature there has recently been a considerable amount of attention devoted to simulation of heat- and mass-transfer processes in porous media. In [2-5], a very general system of differential equations is considered and investigated numerically that describes forced convection of a liquid or gas through a porous body in the absence of thermal equilibrium between the liquid (gas) and solid phases. For this nonequilibrium to be taken into account, a two-phase model of a porous body is used that includes two energy equations (for the liquid and solid phases). Using this model, certain energy characteristics of the process of heat accumulation in a porous layer were investigated numerically in [6], and the difference between the temperatures of phases that appears within the framework of the two-phase model was analyzed in [7].

Many of the analytical works devoted to investigations within the framework of a two-phase model of heating or cooling of porous bodies employ the model of a porous medium suggested by T. Schumann [8]. This model considers the flow of an incompressible liquid (gas) through a porous body and neglects the terms that describe heat transfer by heat conduction in the energy equations for both the liquid (gas) and solid phase. We will follow this model below and make a substantial use of the following assumptions:

- the liquid (gas) phase is incompressible and the flow rate in any transverse cross section of the porous body is constant;
- the thermophysical properties of the liquid (gas) and solid phases are invariable;
- heat transfer and the liquid (gas) flow are one-dimensional;
- heat transfer by heat conduction is negligible in both the solid and liquid (gas) phases.

It follows from [9] that under these assumptions the equations describing the temperature distribution in the liquid and solid phases can be written in the following dimensionless form:

$$\frac{\partial \Theta}{\partial t} = \varphi - \Theta \, , \quad \frac{\partial \varphi}{\partial z} = \Theta - \varphi \, . \tag{1}$$

Here the dimensionless temperatures of the solid and liquid (gas) phases are defined as:

$$\Theta = \frac{T_s - T_1}{T_2 - T_1} \, , \quad \varphi = \frac{T_f - T_1}{T_2 - T_1} \, ,$$

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where T_1 and T_2 are arbitrary constant temperatures selected for reasons of convenient normalization. For example, if $T_s(0, 0) \neq T_f(0, 0)$, then we can select $T_1 = T_s(0, 0)$ and $T_2 = T_f(0, 0)$.

The dimensionless time and coordinate in Eqs. (1) are defined as follows:

$$t = \frac{hat'}{(1-\varepsilon)\rho_s c_s}, \tag{2}$$

$$z = \frac{haz'}{\varepsilon \rho_f c_{pf} v}. \tag{3}$$

The coefficient of heat exchange between the liquid (gas) and the particles of the porous body in Eqs. (2) and (3) can be calculated from a relation given in [10]:

$$h = \left[\frac{d}{\mathrm{Nu}_{fs}\lambda_f} + \frac{d}{\beta\lambda_s}\right]^{-1},$$

where $\beta = 10$, if the particles of the porous body have a spherical shape. The value of the Nusselt number in this formula for $\mathrm{Re}_p > 100$ is well correlated by an expression given in [11]:

$$\mathrm{Nu}_{fs} = \frac{0.255}{\varepsilon}\mathrm{Pr}^{1/3}\mathrm{Re}_p^{2/3}.$$

As follows from [12, 13], the estimates of the quantity Nu_{fs} at small values of Re_p vary between 0.1 and 12.

The specific surface of contact between the solid and liquid (gas) phases, which also enters into Eqs. (2) and (3), can be calculated, according to [14], as $a = 6(1-\varepsilon)/d$.

Analytical solutions of Eqs. (1) under various boundary conditions are contained in [15-19]. A solution for the case when the liquid (gas) temperature on entry into a porous body is dependent on time was obtained in [20] using a Laplace transform. System (1) was solved under the following initial and boundary conditions:

$$\Theta(z, 0) = \Theta_0(z), \quad \varphi(0, t) = \varphi_{\mathrm{in}}(t). \tag{4}$$

After simple transformations, the solution obtained in [20] can be presented in the form

$$\Theta(z, t) = \exp(-z)\int_0^{t-\delta z}\varphi_{\mathrm{in}}(t-\delta z-\tau)\exp(-\tau)I_0(\sqrt{4\tau z})\,d\tau + \exp(\delta z - t)\times$$

$$\times\left[\Theta_0(z) + \int_0^z \Theta_0(z-\xi)\exp(-\xi)\left(\frac{t-\delta z}{\xi}\right)^{1/2} I_1[\sqrt{4\xi(t-\delta z)}]\,d\xi\right], \tag{5}$$

$$\varphi(z, t) = \exp(\delta z - t)\int_0^z \Theta_0(z-\xi)\exp(-\xi)I_0[\sqrt{4\xi(t-\delta z)}]\,d\xi + \exp(-z)\times$$

$$\times\left[\varphi_{\mathrm{in}}(t-\delta z) + \int_0^{t-\delta z}\varphi_{\mathrm{in}}(t-\delta z-\tau)\exp(-\tau)\left(\frac{z}{\tau}\right)^{1/2} I_1(\sqrt{4\tau z})\,d\tau\right], \tag{6}$$

where

$$\delta = \frac{\varepsilon\rho_f c_{pf}}{(1-\varepsilon)\rho_s c_s}. \tag{7}$$

Equations (5)-(7) determine the temperatures of the solid and liquid (gas) phases at the point of the porous body with the coordinate z on arrival of a thermal perturbation at that point, i.e., when $t \geq \delta z$. Since in the model considered the heat transfer by heat conduction is neglected, hitherto the solid phase temperature was remained equal to the initial one prescribed by the function $\Theta_0(z)$ in the first of Eqs. (4).

We consider a one-dimensional porous layer of dimensionless thickness L ($L = haL'/\varepsilon\rho_f c_{pf} v$) whose initial temperature is constant over the thickness. Thereby the dimensionless initial temperature $\Theta(z, 0) = \Theta_0(z) = 0$, if the constants T_1 and T_2 are selected as specified above. Such a choice of initial conditions substantially simplifies Eqs. (5) and (6), since the second term on the right-hand side of Eq. (5) and the first term on the right-hand side of Eq. (6) become identically equal to zero.

Let the dimensionless temperature of the liquid (gas) on entry into the layer at $z = 0$ be prescribed by a certain function of time $\varphi_{in}(t)$. Because of the temperature difference between the phases, the temperature of the liquid (gas) on exit from the layer at $z = L$ will be somewhat higher than the solid phase temperature in the same position. In other words, a portion of the heat that could have been accumulated by the porous layer leaves the layer with the liquid (gas) flow. To increase the efficiency of thermal energy accumulation systems, it is important to find a method for minimizing these heat losses or, in other words, for maximizing the portion of heat accumulated in the porous body.

Now we will set up the following optimal problem. For a prescribed amount of heat E to be transferred by heaters to the liquid (gas) flow and a prescribed duration of the process t_f it is necessary to maximize the amount of heat accumulated by the layer. As a means of control, we will consider the dimensionless temperature of the liquid (gas) on entry into the layer $\varphi_{in}(t)$, which, as assumed, is a certain piecewise-continuous function of time and can vary from a certain minimum value u_{min} corresponding to the liquid (gas) temperature in a "cold faucet" up to a certain maximum value u_{max} corresponding to the liquid (gas) temperature in a "hot faucet."

Mathematically, the given problem can be formulated in the following manner. It is required to find the maximum of the functional

$$\Phi(\varphi_{in}) = \int_0^L \Theta(z, t_f)\, dz \to \max\,, \tag{8}$$

where the function $\Theta(z, t_f)$ is prescribed by Eq. (5) with the following restrictions:

$$\int_0^{t_f} \varphi_{in}(\tau)\, d\tau = E = \text{const}\,, \tag{9}$$

$$u_{min} \leq \varphi_{in}(t) \leq u_{max}\,. \tag{10}$$

In order to reduce problem (8)-(10) to the form of the problem of optimal control, we must transform functional (8). For this, we first write expression (5) for the function $\Theta(z, t)$ at $t = t_f$, transforming it by the following substitution of the integration variable:

$$\tau^* = t_f - \tau - \delta z\,. \tag{11}$$

With allowance for this substitution and with substantial use of the assumption that the initial temperature of the layer is constant (the choice of the constants T_1 and T_2 was specified above), Eq. (5) at $t = t_f$ takes on the form

$$\Theta(z, t_f) = \exp(-z) \int_0^{t_f - \delta z} \varphi_{in}(\tau^*) \exp(-t_f + \tau^* + \delta z) \times$$

$$\times I_0 [\sqrt{4(t_f - \tau^* - \delta z) z}\,]\, d\tau^*\,. \tag{12}$$

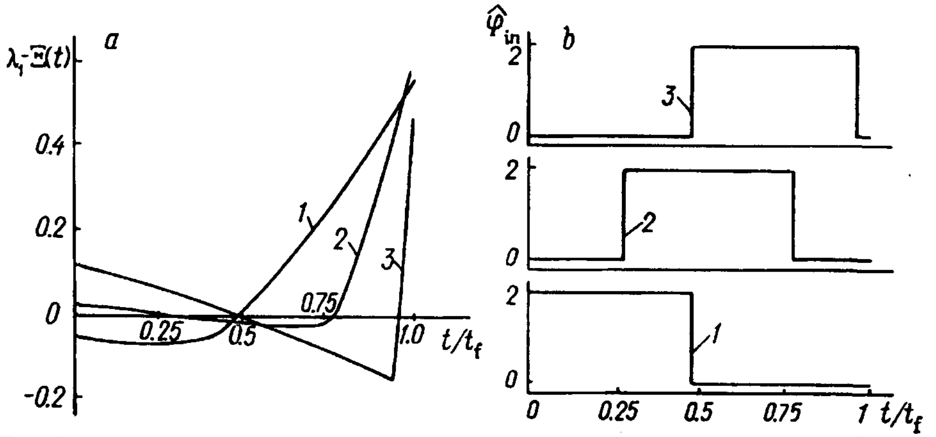

Fig. 1. Graphs of the function $\lambda_1 - \Xi(t)$ and the optimal dependences $\hat{\varphi}_{in}(t)$ (a and b, respectively) for different durations of heating: 1) $t_f = 0.08$; 2) 0.2; 3) 1.

We consider the function

$$\Psi(z, \tau) = \begin{cases} \exp(-z - t_f + \tau + \delta z)\, I_0\, [\sqrt{4(t_f - \tau - \delta z)\, z}\,], & \text{if } 0 \le \tau \le t_f - \delta z, \\ 0, & \text{if } \tau > t_f - \delta z. \end{cases} \tag{13}$$

Then, using Eqs. (12) and (13) and changing the order of integration, we can present functional (8) as

$$\Phi(\varphi_{in}) = \int_0^L \Theta(z, t_f)\, dz = \int_0^{t_f} \varphi_{in}(\tau)\, \Xi(\tau)\, d\tau \to \max, \tag{14}$$

where

$$\Xi(\tau) = \int_0^L \Psi(z, \tau)\, dz.$$

Problem (14), (9)-(10) is a problem of optimal control with isoperimetric condition (9) and with the control region prescribed by inequality (10). This problem can be solved by using the maximum principle (in Hamiltonian form) or the minimum principle (in Lagrangian form) [21-24]. The application of this theorem leads to the following relation, from which the optimal control $\hat{\varphi}_{in}(t)$ can be found:

$$\hat{\varphi}_{in}(t)\, [\lambda_1 - \Xi(t)] \to \min, \tag{15}$$

Condition (15), used with account for restriction (10), allows us to determine the optimal control $\hat{\varphi}_{in}(t)$. Actually, it yields the following relations:

$$\begin{aligned} \hat{\varphi}_{in}(t) &= u_{\min} \quad \text{for} \quad \lambda_1 - \Xi(t) > 0, \\ \hat{\varphi}_{in}(t) &= u_{\max} \quad \text{for} \quad \lambda_1 - \Xi(t) < 0. \end{aligned} \tag{16}$$

The application of relation (16) requires determination of the value of λ_1. For this, it is necessary to solve integral equation (9) numerically with account for Eq. (16).

This problem was solved as follows: a segment was prescribed inside of which the desired value of λ_1 was obviously contained, and then an algorithm was used for finding the root of nonlinear and transcendental equations on the prescribed segment, with Eq. (9) being considered as such.

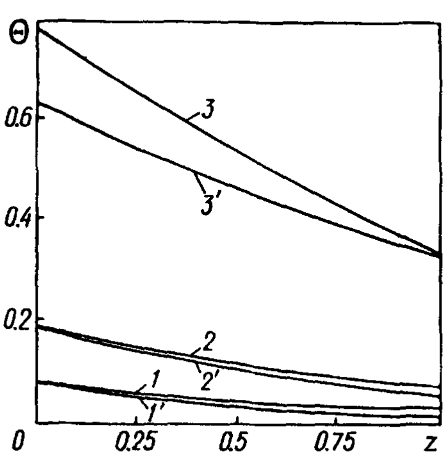

Fig. 2. Temperature distribution in the solid phase on termination of heating at $\varphi_{\text{in}}(t) = \hat{\varphi}_{\text{in}}(t)$ (curves 1, 2, 3) and at $\varphi_{\text{in}}(t) = \varphi^*_{\text{in}}(t)$ (curves 1', 2', 3'): 1, 1') $t_f = 0.08$; 2, 2') 0.2; 3, 3') 1.

Figure 1a shows "function-controller" $\lambda_1 - \Xi(t)$ curves for different durations of the process t_f for the values of the parameters: $u_{\min} = 0$, $u_{\max} = 2$, $E = t_f$, $L = 1$, $\delta = 0.05$, and Fig. 1b presents graphs of the optimal controls $\hat{\varphi}_{\text{in}}(t)$ that correspond to these parameters. It is evident that in the case of a short duration of heating ($t_f = 0.08$) the optimum temperature $\hat{\varphi}_{\text{in}}(t)$ is first equal to the maximum value $u_{\max}$ and then to the minimum one $u_{\min}$. As the heating duration ($t_f = 0.2$) increases, a qualitative change occurs in the behavior of the optimum temperature, i.e., first $\hat{\varphi}_{\text{in}}(t)$ is equal to the minimum value, then to the maximum, and then again to the minimum one. With a further increase in the heating duration ($t_f = 1$), this qualitative behavior persists, however, the third portion (when the optimum temperature is again equal to the minimum one) becomes smaller and smaller.

At the values of the parameters used for calculating Fig. 1, the transition from the first type of optimum temperature behavior (maximum-minimum) to the second type (minimum-maximum-minimum) occurs at $t_f = 0.09913$.

It is of interest to compare the value of the functional $\Phi(\varphi_{\text{in}})$ for the optimum functions shown in Fig. 1b and for the functions $\varphi^*_{\text{in}}(t) = 1$ that correspond to the time-constant inlet temperature of the liquid (gas). It is easily verified that at the values of E used for calculating Fig. 1, the functions $\varphi^*_{\text{in}}(t) = 1$ also satisfy isoperimetric condition (9). The calculations lead to the following relations: when $t_f = 0.08$, $\Phi(\hat{\varphi}_{\text{in}})/\Phi(\varphi^*_{\text{in}}) = 1.320$ (the gain in the amount of accumulated heat in the case of the use of the optimum distribution $\hat{\varphi}_{\text{in}}(t)$ compared to the constant φ^*_{in} is 32.0%); when $t_f = 0.2$, $\Phi(\hat{\varphi}_{\text{in}})/\Phi(\varphi^*_{\text{in}}) = 1.119$ (the gain is equal to 11.9%), and when $t_f = 1$, $\Phi(\hat{\varphi}_{\text{in}})/\Phi(\varphi^*_{\text{in}}) = 1.141$ (the gain equals 14.1%).

Figure 2 illustrates the results of the calculations performed. The figure shows the temperature distributions in the solid phase at $t = t_f$ for the cases when $\varphi_{\text{in}}(t) = \hat{\varphi}_{\text{in}}(t)$ (i.e., with optimal selection of the functions $\varphi_{\text{in}}(t)$) and when $\varphi_{\text{in}}(t) = \varphi^*_{\text{in}}(t)$ (i.e., at $\varphi_{\text{in}}(t) = \text{const}$). These distributions are calculated with the help of Eq. (5). As seen from Fig. 2, the curves corresponding to the solid phase temperature with the optimally selected functions $\varphi_{\text{in}}(t)$ (the graphs of these functions are shown in Fig. 1b) are located above the curves that correspond to the constant inlet temperature of the liquid (gas). Thereby, the choice of the optimal controlling action makes it possible to accumulate a large amount of heat in a porous body. It should be emphasized that this can be attained due to a decrease in the amount of heat that is ejected from a porous layer with the liquid (gas) flow leaving the layer through the boundary $z = L$.

CONCLUSIONS

1. A procedure is suggested for optimizing the process of heating of a porous body from the condition of maximization of the amount of thermal energy accumulated in the body. The control was considered to be exercised by means of the temperature of the liquid (gas) flow on entry into the porous body.

2. It is shown that an increase in the heating duration leads to a qualitative change in the behavior of the optimum temperature of the liquid (gas) on entry into the porous body. At a short duration of heating the optimum temperature is first equal to the maximum value and then to the minimum one. Conversely, for a long duration of heating the optimum temperature is first equal to the minimum value, then to the maximum, and once again to the minimum one. With a further increase in the heating duration, this qualitative behavior persists, however, the third portion (when the optimum temperature is again equal to the minimum one) becomes smaller.

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NOTATION

a, specific surface of contact between the solid and liquid (gas) phases, m^2/m^3; c_p, specific heat at constant pressure, $J/(kg \cdot K)$; d, mean diameter of the porous body particles, m; h, coefficient of heat exchange between the porous body particles and the liquid (gas) phase, $W/(m^2 \cdot K)$; I_ν, modified Bessel function of order ν; L', porous layer thickness, m; L, dimensionless thickness of the porous layer; $Nu_{fs} = hd/\lambda_f$, Nusselt number; $Re_p = \rho_f v d/\mu$, Reynolds number; $Pr = \mu c_{pf}/\lambda_f$, Prandtl number; t', time, sec; t, dimensionless time; t_f, duration of heating; $T(z',t')$, temperature, K; u_{min} and u_{max}, lower and upper boundaries of the range of admissible controls; v, velocity of the liquid (gas) flow, m/sec; z', coordinate, m; z, dimensionless coordinate; δ, parameter; ε, porosity; λ_1, Lagrange multiplier; μ, dynamic viscosity coefficient, $Pa \cdot sec$; $\Theta(z, t)$, dimensionless temperature of the solid phase; $\Theta_0(z)$, the same at $t = 0$; $\varphi(z, t)$, dimensionless temperature of the liquid (gas) phase; $\varphi_{in}(t)$, dimensionless temperature of the liquid (gas) phase entering the porous layer through the boundary $z = 0$; $\varphi_{in}(t)$, optimum inlet temperature; $\varphi^*_{in}(t) = \varphi_{in0} = const$, function that prescribes the time-constant inlet temperature of liquid (gas); ρ, density, kg/m^3. Subscripts: f, liquid (gas) phase; s, solid phase; p, porosity.

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